Advanced Algebra
Algebraic Functions: Graphing Functions with a Transformational Approach

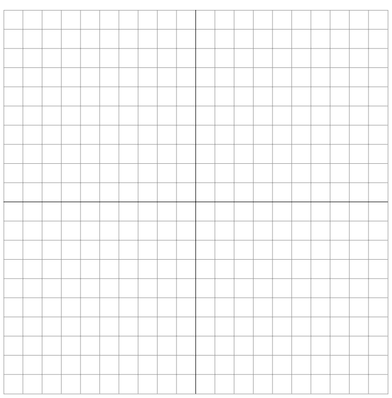
I. Parent Functions

Even the most complicated functions can be categorized by a ***parent function***. If you can identify the parent function and its shape, you are half-way to graphing the more complicated function. For example:

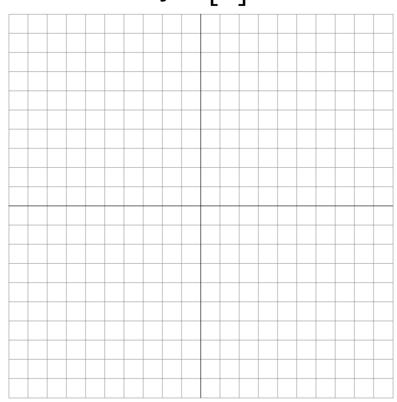
Equation	Parent
$y = 3x - 5$	$y = x$
$y = 2x^2 - 5$	$y = x^2$
$y = (x + 1)^3 + 2$	$y = x^3$
$y = 2\sqrt{x-1} + 1$	$y = \sqrt{x}$
$y = 3x^2 - 3x + 4$	$y = x^2$

Carefully graph each of the following ***parent functions***. Be sure to label all important points (i.e. vertices, asymptotes, intercepts, slope, etc.) and some other key points.

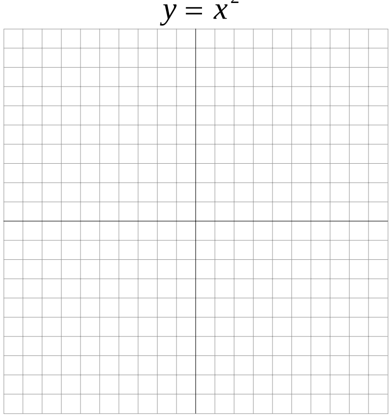
$y = |x|$



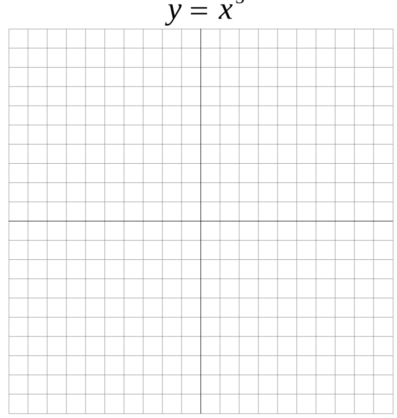
$y = [x]$



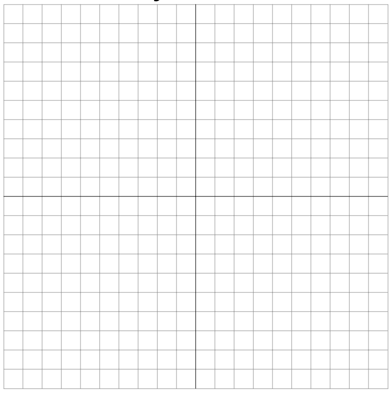
$y = x^2$



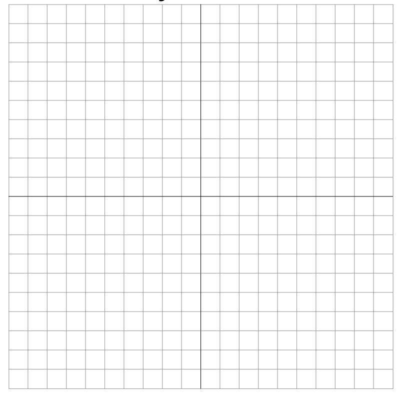
$y = x^3$



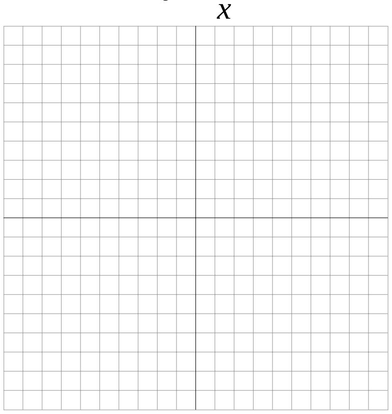
$y = \sqrt{x}$



$y = \sqrt[3]{x}$



$y = \dfrac{1}{x}$



II. Transforming Parent Functions

Here we will look some parent functions that have had coefficients and constants multiplied and added to them. We will use the following model to represent the parent function, $y = f(x)$, with transformations applied to it:

$$y = a \cdot f(bx + c) + d$$

The new aspects of the function, *a*, *b*, *c* and *d* are coefficients and constants and they effect the graph. The transformed graph will sometimes have a different slope and be centered in a difference place on the coordinate plane than the parent graph was, *but the general shape will be the same*.

Consider, for example, $y = 4(3x - 1)^3 + 2$.

‘a’ will refer to any number that is multiplied by the function, here, a = 4
‘b’ will refer to any number that is multiplied by the variable, here, b = 3
‘c’ will refer to any number that is added to the variable, here, c = -1
‘d’ will refer to any number that is added to the function, here, d = 2

For each of the following,

a) state the *parent* function, and
b) state the values of *a*, *b*, *c and d*:

1) $y = 2|3x - 5| - 7$

2) $y = -2(x - 3)^2 - 7$

3) $y = 3 - \sqrt[3]{2x}$

4) $y = 3x^2 - 4$

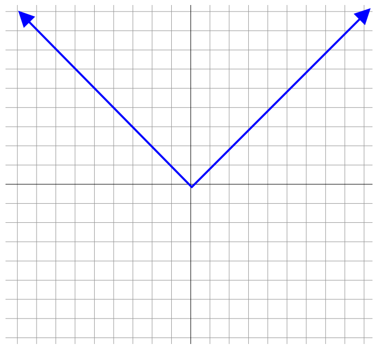
5) $y = \sqrt{2x + 5} - 3$

6) $y = (5x - 1)^2 + 3$

To graph these transformed functions, we will use a 3-Step Approach. In this example we will graph the function:

$$y = -3|2x - 5| + 4$$

Step 1. **Graph the *parent* curve,** $y = |x|$ - make a table of key points



x	y
-1	1
0	0
1	1

Step 2. **Identify the coefficients, *a* and *b*.**

In the example, $y = -3|2x - 5| + 4$, a = -3 and b = 2

Step 3. **Graph the parent curve as it is affected by coefficients *a* and *b*.**
To do this, take the list of ordered pairs on the parent graph, for each *x*-value in the list, divide *x* by 'b' and for each *y*-value, multiply *y* by 'a'.

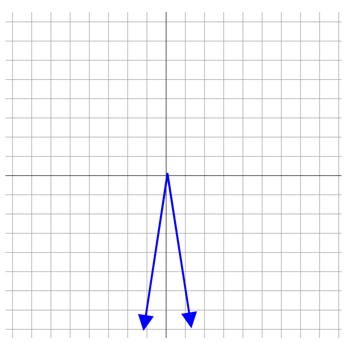
$$y = -3|2x - 5| + 4\text{ , a = -3, b = 2}$$

Pts on Parent

x	y
-1	1
0	0
1	1

Pts on New Graph: *(x ÷ 2, y • -3)*

x	y
$-\frac{1}{2}$	-3
0	0
$\frac{1}{2}$	-3



Note that coefficients cause ***dilations*** (they stretch the graph) and ***reflections*** (they flip the graph).

Step 4. **Identify the constants, *c* and *d*.**

In the example, $y = -3|2x - 5| + 4$, c = -5 and d = 4

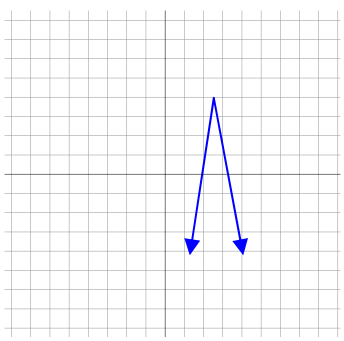
Step 5. **Graph the curve from step 3 as it is affected by the constants *c* and *d*.**
To do this, take the list of ordered pairs from the last step. Then, for each *x*-value in the list, add $-c/b$ and for each *y*-value in the list, add ‘*d*’.

Pts from Step 2

x	y
$-\frac{1}{2}$	-3
0	0
$\frac{1}{2}$	-3

Pts on New Graph: $\left(x + -\left(-\frac{5}{2}\right), y + 4\right)$

x	y
2	1
$\frac{5}{2}$	4
3	1



Note that constants cause ***translations*** (they slide the graph to a new position).