Inverse Relations

and Functions

Recall: Relation vs. Function

Relation: any set of ordered pairs

Function: A relation such that all domain values have a unique range value

Inverse Relations

If R is any relation then R^{-1} is its inverse if for every point (x, y) in R, (y, x) is in R^{-1}.

*switch x and y of each point!

The domain and range are switched

If R is a function, R^{-1} does not have to be a function

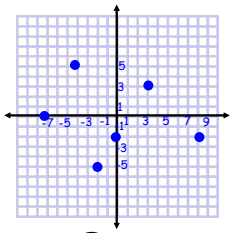
R = {(3, -2), (-1, 0), (4, 6)}

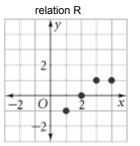
R^{-1} =

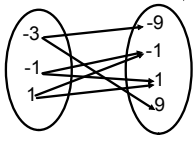
Find the inverse. State whether either is a function or not.

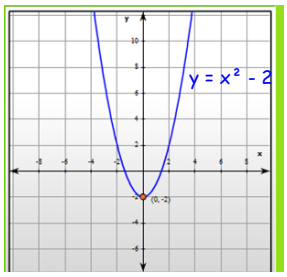
Relation s

x	1	2	3	4
y	−1	0	1	1









$y = x^2 - 2$

Graph its inverse by finding points.

y		y^{-1}	
x	y	x	y

$(f \circ f^{-1})(x) = x$ and $(f^{-1} \circ f)(x) = x$

the composition of a function and its inverse is always x!

Let $f(x) = 2x + 5$. Find each value.

27. $(f^{-1} \circ f)(-1)$

28. $(f \circ f^{-1})(3)$

Show that the two functions are inverses:

$f(x) = 2x - 5 \qquad g(x) = \frac{1}{2}x + \frac{5}{2}$

Show that the two functions are inverses:

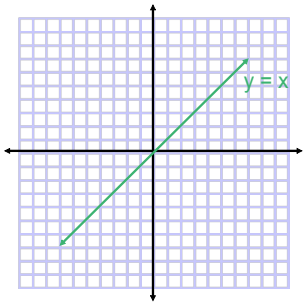
$f(x) = (x+1)^3 - 3 \qquad g(x) = \sqrt[3]{x+3} - 1$

Show the two functions are inverses

1. $y = 2x - 8$ and $y = \frac{1}{2}x + 4$

2. $y = x^2 - 4x$ and $y = \sqrt{x+4} + 2$ $(x > -4)$

The graphs of inverses will be a reflection over the identity line, y = x.



Graph:

$f(x) = (x+1)^3 - 3$

$g(x) = \sqrt[3]{x+3} - 1$

You can find the inverse of an equation in x and y by switching the x & y in the equation, solve for the "new" y.

Don't forget to use ± when taking the square root of both sides.

1. $y = 2x + 6$

2. $y = x^2 - 2$

The inverse of a horizontal line y = b is a vertical line x = b

Find the inverse of each function. Is the inverse a function?

$y = 3(x + 1)$

$y = -x^2 - 3$

$y = (x + 4)^2 - 4$

For each function f, find f^{-1} and the domain and range of f and f^{-1}. Determine whether f^{-1} is a function.

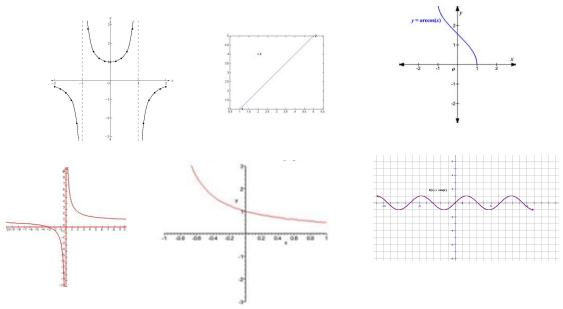
$f(x) = -\frac{1}{5}x + 2$

$f(x) = x^2 + 4$

$f(x) = \sqrt{x - 1}$

$f(x) = \sqrt[3]{x+2} - 5$

When will a function have an inverse that is also a function?



If a function is not 1-1, but you want the inverse to also be a function, then

Restrict the domain

To the largest interval that is 1-1.

$y = x^2$ is 1-1 on the interval $[0,\infty)$ so an inverse function would be $y = \sqrt{x}$

or use the interval $(-\infty, 0]$ so the inverse function is $y = -\sqrt{x}$

On what interval would $y = (x + 2)^2 - 4$ have an inverse function? What is the function?