Logarithms

$$b^x = n \longleftrightarrow log_b n = x$$

Exponent Rules		Logarithm Rules
$x^a \cdot x^b = x^{a+b}$	⟷	$\log_x a + \log_x b = \log_x ab$
$\frac{x^a}{x^b} = x^{a-b}$	⟷	$\log_x a - \log_x b = \log_x \frac{a}{b}$
$(x^a)^b = x^{ab}$	⟷	$\log_x a^b = b\log_x a$
$x^{-a} = \frac{1}{x^a}$	⟷	$\log_x \frac{1}{a} = -\log_x a$
$x^{\frac{1}{a}} = \sqrt[a]{x}$	⟷	$\log_x \sqrt[a]{x} = \frac{1}{a}$
$x^0 = 1$	⟷	$\log_x 1 = 0$

Special Logs

Common Log: $\log x = \log_{10} x$

Natural Log: $\ln x = \log_e x$

Change of Base Formula

$$\log_x a = \frac{\log_n a}{\log_n x} = \frac{\log a}{\log x} = \frac{\ln a}{\ln x}$$

Examples

1. $\log_2 64 = 6$ because $2^6 = 64$

2. $\log_3 9 = 2$ because $3^2 = 9$

3. $\log_3 \frac{1}{9} = -2$ because $3^{-2} = \frac{1}{9}$

4. $\log_{\frac{1}{3}} 9 = -2$ because $\frac{1}{3}^{-2} = 9$

5. $\log_2 8 = 3$ because $2^3 = 8$

6. $\log_8 2 = \frac{1}{3}$ because $8^{\frac{1}{3}} = \sqrt[3]{8} = 2$

7. $\ln e^{4x} = \log_e e^{4x} = 4x$

8. $\log_4 8 = \log_4 (2 \cdot 4) = \log_4 2 + \log_4 4 = \frac{1}{2} + 1 = \frac{3}{2}$

9. $\log_4 8 = \log_4 (16 \div 2) = \log_4 16 - \log_4 2 = 2 - \frac{1}{2} = \frac{3}{2}$

10. $\log_4 8 = \log_4 (2^3) = 3\log_4 2 = 3\left(\frac{1}{2}\right) = \frac{3}{2}$

Compound Interest

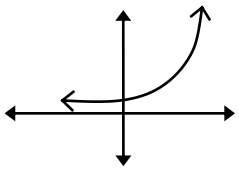
$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$A = Pe^{rt}$$

Inverses

$$b^{\log_b x} = x$$

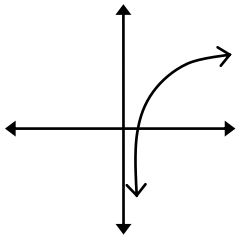
$$\log_b b^x = x$$



$f(x) = b^x$

D: $(-\infty, \infty)$
R: $(0, \infty)$

Exponential



$f(x) = log_b x$

D: $(0, \infty)$
R: $(-\infty, \infty)$

Logarithmic